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DESIGN OF AIRFOILS PROVIDING THE ABSENCE OF SEPARATION IN A COMPRESSIBLE FLOW IN A SPECIFIED RANGE OF ANGLES OF ATTACK

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A problem of modification of the classical airfoils that ensure the absence of separation in a subsonic ideal-gas flow in a specified range of angles of attack is solved by a numerical–analytical method based on the quasi-solution of inverse boundary-value problems of aerohydrodynamics and Kármán–Jiang formulas. Loitsyanskii's criterion of the non-separated flow is used to determine the boundary-layer separation point.

Key words: *inverse boundary-layer problem of aerohydrodynamics, method of quasi-solutions, non-separated flow, allowance for compressibility.*

Introduction. Problems of design and modification of airfoils often involve methods that imply a goal-oriented search and correction of the airfoil shape by solving an inverse boundary-value problem of aerohydrodynamics. The essence of the problem is determining the airfoil shape on the basis of a pressure or velocity distribution of a liquid or gas flow on the airfoil contour, which ensures the necessary aerodynamic characteristics: maximum lift coefficient, minimum drag of the airfoil, maximum lift-to-drag ratio, non-separated flow, etc. A numerical–analytical solution of the problem of constructing airfoils providing a non-separated flow in an ideal incompressible fluid for a given angle of attack was obtained in [1].

The present paper describes the solution of the problem of modification of the classical airfoils, in particular, the Joukowski and Clark airfoils, aimed at providing a non-separated flow around the airfoil in a specified range of angles of attack with allowance for flow compressibility at Mach numbers $\mathrm{M}_\infty < 1$.

Formulation of the Problem. A nonpermeable airfoil is placed in the plane $z = x + iy$ at an angle of attack α in a planar steady ideal-gas flow with a specified Mach number at infinity M_∞ (Fig. 1a). The contour L_z of the airfoil with a perimeter L is closed and smooth everywhere, except for the trailing edge B, where the internal (with respect to the flow domain) angle is 2π. The origin of the coordinate system used coincides with the point B, and the abscissa axis is aligned parallel to the direction of flow velocity at infinity. The arc coordinate s of the contour L_z is counted from $s = 0$ at the point B to $s = L$ at the same point, so that the flow domain remains on the left with increasing s along L_z. The range of angles of attack $[\alpha_1^*, \alpha_2^*]$ without flow separation from this airfoil is known. The Reynolds number at infinity Re_∞ is specified.

The task is to modify the airfoil to ensure a non-separated flow in a greater range of angles of attack $[\alpha_1, \alpha_2]$, where $\alpha_1 < \alpha_1^*$ and $\alpha_2 > \alpha_2^*$, to calculate the aerodynamic characteristics of this airfoil and compare them with the characteristics of the initial airfoil, to find the distribution of normalized velocity $\lambda = \lambda(s)$, $s \in [0, L]$, on the contour of the modified airfoil at angles of attack α_1 and α_2, and to verify the non-separated character of the flow past this airfoil by means of solving a direct problem by the Fluent software package.

Models and Methods. The model is constructed for a purely turbulent flow where the arc abscissa of the point s_{tj} of the laminar–turbulent transition of the boundary layer coincides with the point s_* of flow branching (the subscripts $j = 1$ and $j = 2$ refer to the upper and lower surfaces of the airfoil, respectively). The point s_s of

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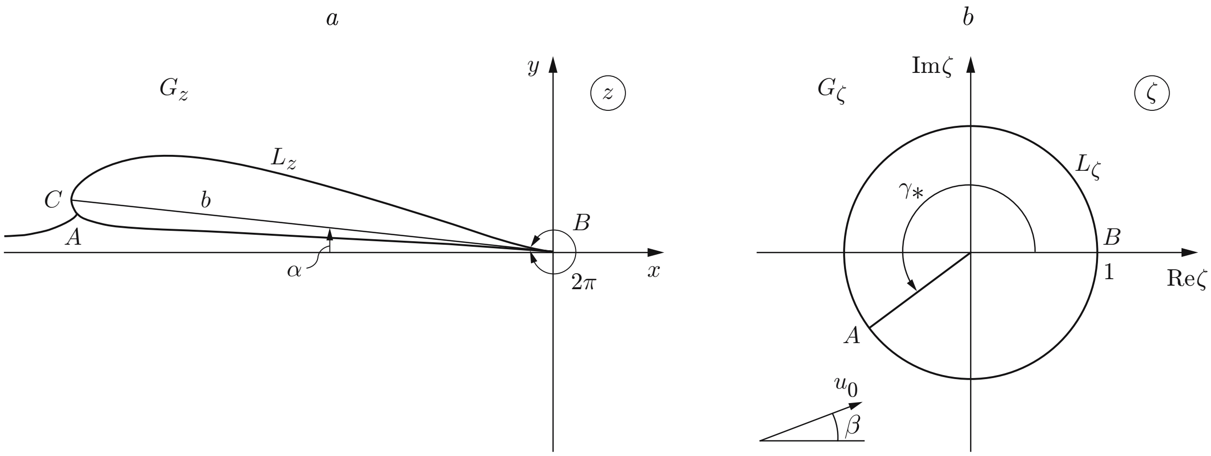

Fig. 1. Layout of the problem: (a) specified airfoil; (b) canonical domain.

separation of the turbulent boundary layer was determined on the basis of Loitsyanskii's criterion of a non-separated flow [2] in the form

$$f(s) \geqslant f_0, \qquad f_0 = -2, \tag{1}$$

where $f(s)$ is the shape factor,

$$f(s) = \frac{a\lambda'(s)}{|\lambda(s)|^b}\Big|\int\limits_{s_{tj}}^{s} |\lambda(\tau)|^{b-1}\,d\tau\Big|,$$

$a = 1.17$ and $b = 4.57$ are empirical constants, and λ_{tj} is the absolute value of velocity at the points s_{tj}.

Kármán–Jiang formulas [3] allow the distribution of the pressure coefficient on the airfoil contour in an incompressible fluid to be recalculated to a gas flow with an arbitrary free-stream Mach number $\mathrm{M}_\infty < 1$ and an unchanged angle of attack. The dependence between the velocities $V(s)$ and $\lambda(s)$ on the airfoil contour in an incompressible flow and a compressible flow is expressed by the formula

$$V(\lambda) = \begin{cases} V_\infty\Big(\dfrac{1-(1-\mathrm{M}_\infty^2)^{1/2}c_p}{1-[1-(1-\mathrm{M}_\infty^2)^{1/2}]c_p/2}\Big)^{1/2}, & |\lambda| \geqslant \lambda_\infty, \\ 2\lambda[1+(1+4c^2\lambda^2)^{1/2}]^{-1}, & |\lambda| < \lambda_\infty, \end{cases} \tag{2}$$

where c_p is the pressure coefficient in the gas:

$$c_p = \frac{2}{k\mathrm{M}_\infty^2}\Big[\Big(\frac{1-\lambda^2/h^2}{1-\lambda_\infty^2/h^2}\Big)^{k/(k-1)} - 1\Big],$$

$c^2 = 0.296$, $k = 1.41$, and $h^2 = (k+1)/(k-1)$.

The result of recalculation is the velocity distribution on the sought airfoil in an incompressible fluid flow moving with a velocity V_∞. The airfoil shape can be found by solving an inverse boundary-value problem of aerohydrodynamics with the use of the model of an ideal incompressible fluid and the method of quasi-solutions to satisfy the solvability conditions [4]. The distribution of the normalized velocity, which is changed in constructing the quasi-solution, can be reconstructed by the formulas

$$\lambda(V) = \begin{cases} V/(1-c^2V^2), & V \leqslant V_\infty, \\ h[1-(1-\lambda_\infty^2/h^2)/(kM_\infty^2 c_p/2+1)^{(1-k)/k}]^{1/2}, & V \geqslant V_\infty, \end{cases} \tag{3}$$

where $c_p = c_p^0\{(1-\mathrm{M}_\infty^2)^{1/2} + [1-(1-\mathrm{M}_\infty^2)^{1/2}]c_p^0/2\}^{-1}$, where $c_p^0 = 1-(V/V_\infty)^2$ is the pressure coefficient in an incompressible fluid.

Constructing a non-separated velocity distribution on the diffuser segment is based on specifying the distribution of the shape factor $f(s)$. Obviously, if the function $f(s)$ satisfies the criterion of a non-separated flow (1), then the corresponding function $\lambda(s)$ also satisfies this criterion.

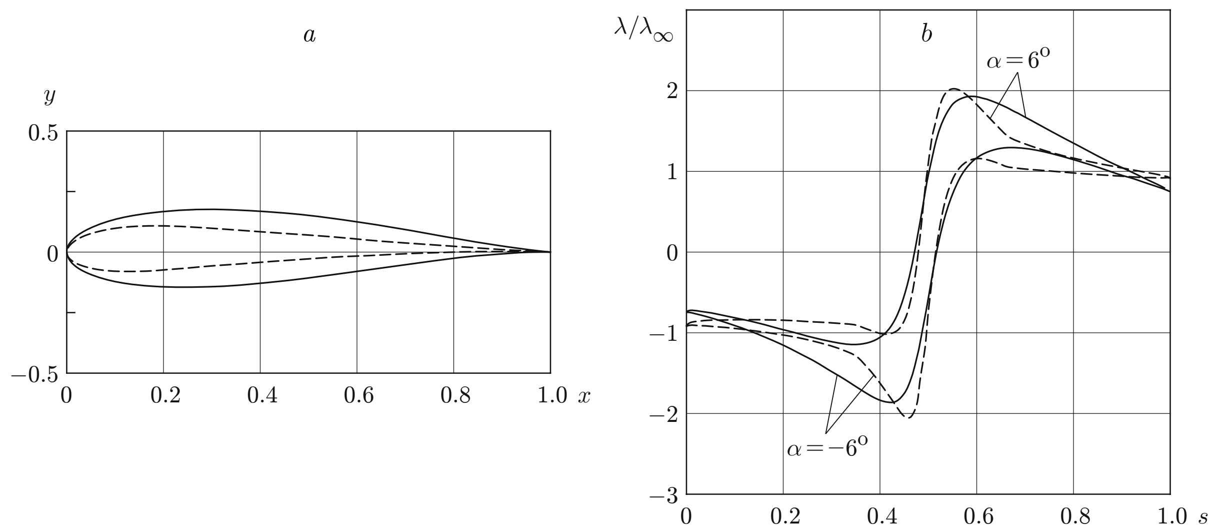

Fig. 2. Modification of the Joukowski airfoil: (a) initial and modified contours; (b) distribution of dimensionless velocity on the airfoils; the solid and dashed curves refer to the initial and modified airfoils, respectively.

The velocity distribution on the diffuser segment is replaced by the non-separated distribution in the following manner. We introduce a dimensionless coordinate $\sigma = (s - s_*)/(L - s_*)$, $\sigma \in [0, 1]$. It is known that $\lambda = \lambda(\sigma)$ and $\lambda(\sigma_s) = \lambda_{\max}$ on the segment $[0, \sigma_s]$, where σ_s is the point of flow separation. Then, beginning from the point σ_s, the velocity distribution on the segment $[\sigma_s, 1]$ can be further constructed by the formula [4, § 39]

$$\lambda(\sigma) = \lambda_{\max} \exp\Big(a^{-1} \int\limits_{\sigma_s}^{\sigma} g(\tau)[Q(\tau)]^{-1}\, d\tau\Big), \tag{4}$$

where $Q(\sigma) = \int\limits_{\sigma s}^{\sigma} [1 - (b-1)g(\tau)/a]\, d\tau$, and the function $g = g(\sigma) = -2$ corresponds to a specified law of shape factor variation.

For a specified distribution of incompressible flow velocity $V(s)$ on the sought airfoil contour L_z, we solve an inverse boundary-value problem of aerohydrodynamics with the use of the quasi-solution method. A function that ensures conformal mapping of the canonical domain G_ζ (exterior of a circle of unit radius; see Fig. 1b) with a boundary L_ζ onto the exterior of the sought airfoil has the form

$$z(\zeta) = u_0 \exp(-i\beta) \int\limits_1^{\zeta} \exp(-\tilde{\chi}(\zeta))(1 - 1/\zeta)\, d\zeta, \tag{5}$$

where $u_0 = \Gamma/(4\pi \sin\beta)$ is the flow velocity at infinity in the plane ζ, $\Gamma = \int\limits_0^L V(s)\, ds$ is the velocity circulation over the airfoil contour, β is the theoretical angle of attack determined from the equation $\beta + \cot\beta = \pi\varphi_1/\Gamma - \pi/2$, $\varphi_1 = \int\limits_{s_*}^{L} V(s)\, ds$, $\tilde{\chi}(\zeta)$ is a function, which is analytical in the domain G_ζ and continuous in the closed domain $\overline{G}_\zeta$:

$$\tilde{\chi}(\zeta) = \tilde{S} - i\tilde{\theta} = \chi(\zeta) - \chi_0(\zeta) = -\frac{1}{2\pi} \int\limits_0^{2\pi} \tilde{S}(\tau)\, \frac{\exp(i\tau) + \zeta}{\exp(i\tau) - \zeta}\, d\tau,$$

$\chi(\zeta) = \ln(dw/dz) = S - i\theta$ is the Joukowski–Mitchell function, $\chi_0(\zeta) = \ln(1 - \zeta_*/\zeta)$, $\zeta_* = \exp(i\gamma_*)$ is the image of the stagnation point A on the boundary L_ζ, and γ_* is the angular coordinate. The real part of the function $\tilde{\chi}(\zeta)$ on the contour L_ζ is known:

$$\tilde{S}(\gamma) = \operatorname{Re}\tilde{\chi}(\exp(i\gamma)) = \ln|V(s(\gamma))/[2\sin((\gamma - \gamma_*)/2)]|,$$

and the imaginary part on L_ζ is found by the formula

$$\tilde{\theta}(\gamma) = -\frac{1}{2\pi}\int_0^{2\pi} \tilde{S}(\tau)\cot\frac{\tau-\gamma}{2}\,d\tau.$$

Parametric equations of the sought airfoil contour L_z are obtained by the transition in Eq. (5) to the limiting values at $\zeta = \exp(i\gamma)$, where $0 \leqslant \gamma \leqslant 2\pi$.

The conditions of solvability of the inverse boundary-value problem of aerohydrodynamics are understood as the conditions of closedness of the sought contour L_z and the condition of coincidence of the specified value of velocity at infinity with the value determined in the course of the solution.

After the replacement of the velocity distribution over the airfoil contour L_z by a non-separated distribution by Eq. (4) for the modified airfoil, the velocities on the trailing edge B being approached over the upper and lower surfaces can differ from each other, i.e., $|\lambda(0)| \neq |\lambda(L)|$. As a result, the point B acquires a singularity, namely, a logarithmic spiral. To eliminate this singularity in solving the inverse problem of aerohydrodynamics by the quasi-solution method, the analytical function $\tilde{\chi}(\zeta)$ has to be supplemented by the function $\Delta\chi(\zeta) = (im/\pi)\ln(1-1/\zeta)$, where $m = \mathrm{Re}\,\tilde{\chi}(0) - \mathrm{Re}\,\tilde{\chi}(2\pi)$. The resultant quasi-solution

$$\tilde{\chi}_1(\zeta) = \tilde{\chi}(\zeta) - \Delta\chi(\zeta) \tag{6}$$

is substituted into Eq. (5), and then an airfoil contour is constructed, which not only satisfies the conditions of solvability of the inverse boundary-value problem of aerohydrodynamics but also has coincident values of velocity on the trailing edge B: $|\lambda_1(0)| = |\lambda_1(L)|$ [$\lambda_1(s)$ is the velocity on the airfoil contour predicted by the quasi-solution].

Iterative Method of the Solution. With allowance for the considerations given above, the construction of an airfoil providing a non-separated compressible flow in a given range of angles of attack can be reduced to the following iterative process.

We solve a direct boundary-value problem of aerohydrodynamics for the initial airfoil in a compressible flow at an angle of attack α_2 and find the distribution of the dimensionless velocity $\lambda = \lambda(s)$, $s \in [0, L]$, on the contour of this airfoil. The criterion of a non-separated flow (1) is checked for the distribution of the dimensionless velocity $\lambda(s)$ at the angle of attack α_2. If this criterion is not satisfied, then $\lambda(s)$ is changed in accordance with Eq. (4) with allowance for the condition of a non-separated flow, beginning from the flow-separation point s_s. After that, the velocity distribution $\lambda(s)$ is recalculated for an ideal incompressible fluid by Eqs. (2), the airfoil is turned at an angle of attack α_1, and Eq. (3) is applied to find the distribution $\lambda(s)$ for α_1. Satisfaction of the criterion of a non-separated flow (1) for $\lambda(s)$ at α_1 is checked. If the criterion is not satisfied, then the distribution $\lambda(s)$ is changed by Eq. (4) to a non-separated distribution. After that, the airfoil shape is found by solving an inverse boundary-value problem of aerohydrodynamics by using quasi-solution (6). If the distribution $\lambda(s)$ changes so that flow separation occurs, it is necessary to return to the beginning of the iterative process. If no separation is observed at the angles of attack α_1 and α_2, the modification is finished and the problem posed is solved.

Results of Numerical Calculations. The initial airfoil to be subjected to modifications was the Joukowski airfoil whose contour is shown in Fig. 2a by the solid curve (hereinafter, the coordinates of the airfoil contour are normalized to the airfoil chord b, the arc abscissas are normalized to the perimeter L of the contour L_z, and the velocity distributions $\lambda(s)$ are normalized to the velocity λ_∞). The airfoil provides a non-separated compressible flow only in the range of angles of attack $\alpha \in [-2^\circ, 2^\circ]$.

The result of modification with the above-described iterative method in the range of angles of attack $\alpha \in [-6^\circ, 6^\circ]$ is illustrated by the dashed curve in Fig. 2a. The corresponding distributions of dimensionless velocity on the contours of the initial and modified airfoils are plotted in Fig. 2b.

For verification of the results obtained, we solved a direct problem by the Fluent software package. The calculations for the direct problem were performed with the Spalart–Allmaras one-parameter model of turbulence. Discretization of the flow domain was performed by a structured grid with rectangular cells. Domain discretization at infinity was made as a C-type grid nested into an O-type grid. The grid had the following size: 10 airfoil chords in the upstream direction, 25 airfoil chords in the downstream direction, and 10 airfoil chords upward and downward with respect to the airfoil. No-slip conditions were set on the airfoil contour. The flow at infinity had the following characteristics: Mach number $\mathrm{M}_\infty = 0.5$, Reynolds number $\mathrm{Re}_\infty = 10^7$, density of air $\rho = 1.29$ kg/m^3, pressure $p_\infty = 101,325$ Pa, temperature $T_\infty = 273$ K, and dynamic viscosity $\nu = 1.78 \cdot 10^{-5}$ m^2/sec.

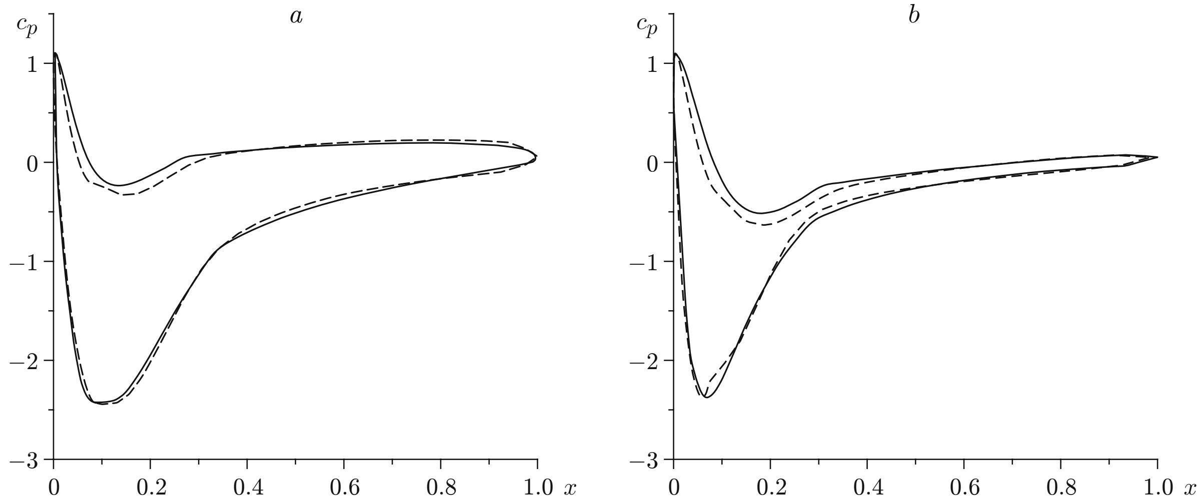

Fig. 3. Distribution of the pressure coefficient over the contour of the modified Joukowski airfoil, obtained by numerical simulations (solid curves) and by the Fluent software package (dashed curves): (a) $\alpha = 6^\circ$; (b) $\alpha = -6^\circ$.

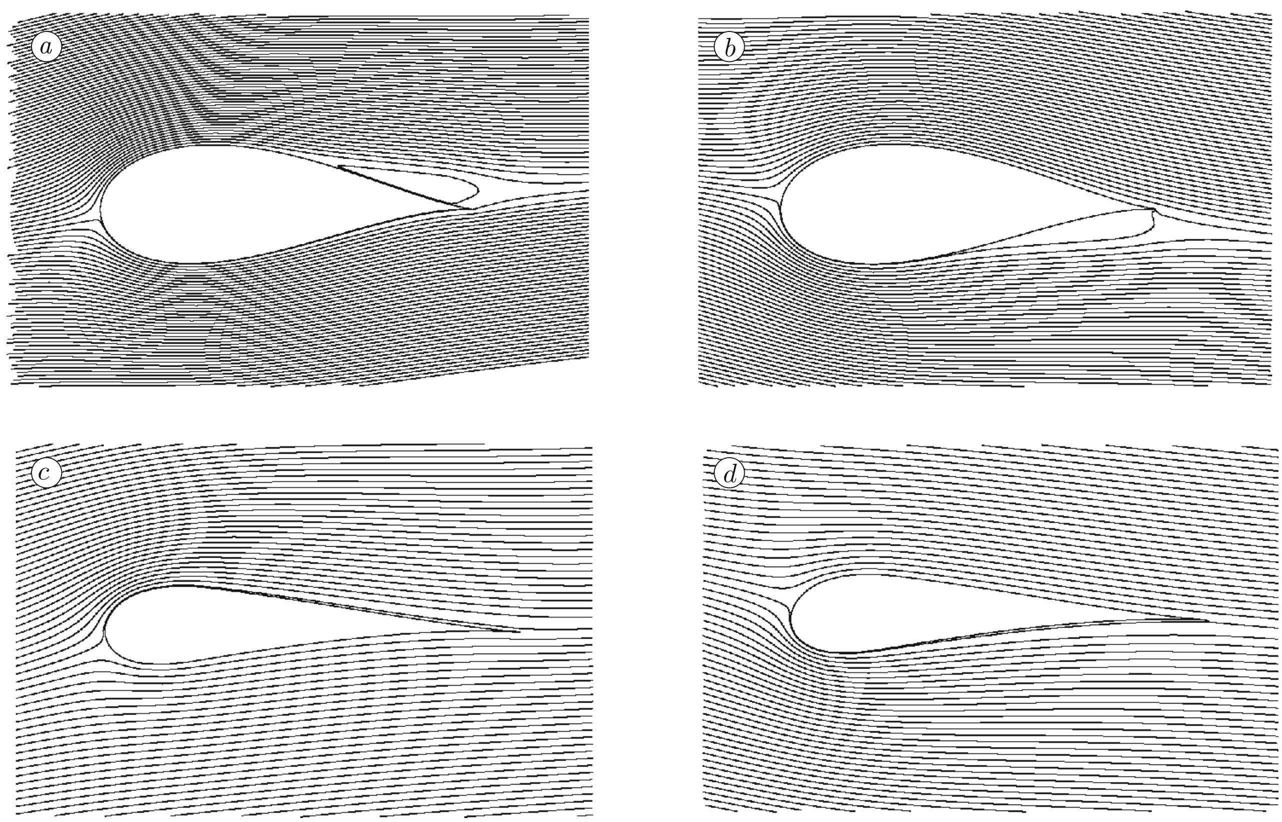

Fig. 4. Flow patterns around the initial (a and b) and modified (c and d) contours of the Joukowski airfoil, obtained by the Fluent software package: $\alpha = 6^\circ$ (a and c) and -6° (b and d).

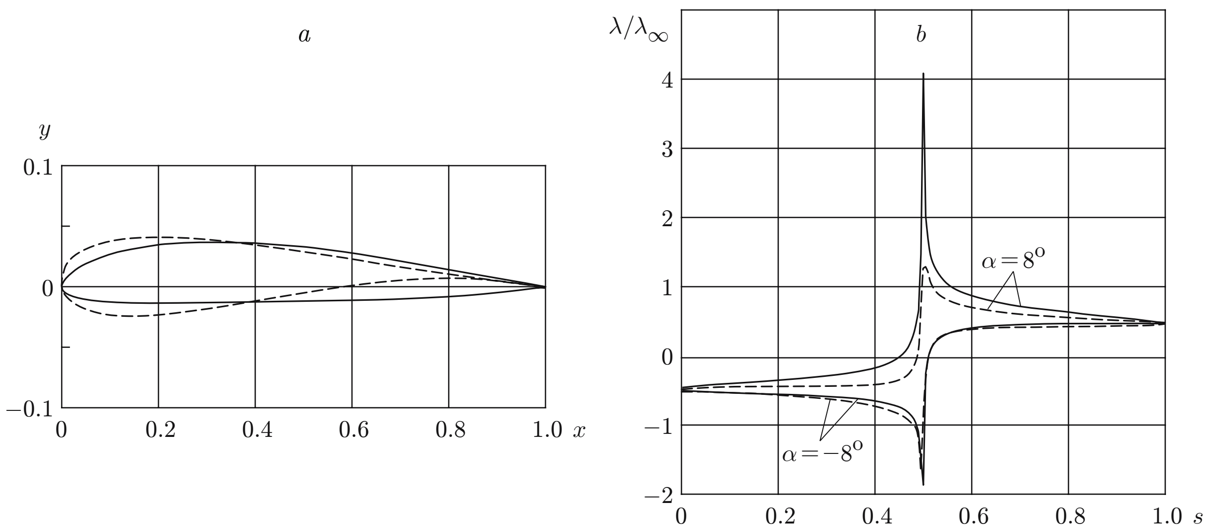

Fig. 5. Modification of the Clark-05 airfoil: (a) initial and modified contours; (b) distribution of dimensionless velocity on the contours; the solid and dashed curves refer to the initial and modified airfoils, respectively.

TABLE 1

Aerodynamic Characteristics of Initial and Modified Airfoils

Airfoil	α, deg	Numerical calculation	Calculation by the Fluent software package		
		C_y	C_y	C_x	K
Joukowski airfoil:					
initial	6	0.678	0.556	0.124	4.48
	−6	−0.152	−0.161	0.117	−1.40
modified	6	0.922	0.896	0.096	9.33
	−6	−0.450	−0.435	0.085	−5.06
Clark-05:					
initial	8	0.845	0.837	0.104	8.04
	−8	−0.658	−0.726	0.083	−8.74
modified	8	0.987	0.939	0.095	9.88
	−8	−0.702	−0.739	0.078	−9.17

The distributions of the pressure coefficient for the modified airfoil at angles of attack $\alpha = 6°$ and $\alpha = -6°$ are plotted in Fig. 3. Figure 4 shows the flow pattern around the initial and modified contours of the Joukowski airfoil. It is seen that the initial airfoil at angles of attack $\alpha = 6°$ and $\alpha = -6°$ provides flow separation, whereas the flow around the modified airfoil is non-separated.

Another example taken for modification was the Clark-05 airfoil [5] whose contour is shown by the solid curve in Fig. 5a. This airfoil provides a non-separated compressible flow in the range of angles of attack $\alpha \in [-5°, 5°]$. The result of its modification in the range of angles of attack $\alpha \in [-8°, 8°]$ is shown by the dashed curve in Fig. 5a. The corresponding distributions of dimensionless velocity on the initial and modified contours of the Clark-05 airfoil are plotted in Fig. 5b. For verification of the results obtained, a direct problem was again solved by the Fluent software package.

The aerodynamic characteristics of the initial and modified airfoils are summarized in Table 1. Numerical calculations based on the iterative method described above show that the lift coefficient C_y of the modified airfoils is greater for positive angles of attack and smaller for negative angles of attack than the corresponding values for the initial airfoils. A computational experiment performed with the use of the Fluent software package confirmed these conclusions. An analysis of results of the computational experiment showed that the drag coefficient C_x of the modified airfoils in the range of angles of attack examined is lower than the drag coefficient of the initial airfoils, whereas the lift-to-drag ratio K of the modified airfoils is higher.

Conclusions. Thus, the proposed method allows both the classical airfoils and any other airfoils to be modified in order to ensure a non-separated compressible flow in a greater range of angles of attack. In addition, the results of computational experiments show that the modified airfoils have a lower drag coefficient than the initial airfoils and, hence, a higher lift-to-drag ratio.

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